Particle physics: the flavour frontiers

Lecture 3: Non-Abelian symmetries and spontaneous symmetry breaking

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Short recap

<u>Last time we discussed</u>

- Different types of Abelian internal symmetries
- What happens if we impose these symmetries on the Lagrangian and what is the allowed spectrum
- An example of an Abelian theory: Quantum Electrodynamics (QED)

Today's learning targets

<u>Today you will …</u>

- get familiar with different types of non-Abelian internal symmetries and how they give rise to vector boson self-interactions
- learn what happens if we impose these symmetries on the Lagrangian and what is the allowed spectrum
- learn about the mechanism of Spontaneous Symmetry Breaking

Non-Abelian symmetries

- **Noncommutative symmetries:** the result of applying two symmetry transformations might depend on the order in which they are applied
- In the SM both *Abelian* (commutative) and *non-Abelian* (noncommutative) symmetries play a role
- We are interested in internal symmetries where the symmetry transformation is unitary

$$\phi \to U\phi, \qquad UU^\dagger = U^\dagger U = \mathbf{1}$$

- Most relevant for the SM are $U(1)$ (Abelian), $SU(2)$ (non-Abelian) and $SU(3)$ (non-Abelian)
- What are the main differences between $U(1)$ and $SU(N)$?

Non-Abelian symmetries

$U(1)$ symmetry

- Complex field ϕ with $q \neq 0$
- ϕ transforms as: $\phi \rightarrow e^{iq\theta}\phi$ (q –real number)
- Transformation law is defined for each single field separately (phase change proportional to q)
- $U(1)$ *invariance of* $\mathcal{L}$: each term must consist of a product of fields such that the sum of their charges is zero

$SU(N)$ symmetry

- Field ϕ in a representation R of dimensions $M > 1$
  - ϕ is a vector with M components: ϕ_i $(i = 1, \dots, M)$
- ϕ transforms as: $\phi_i \rightarrow \left(e^{iT_a\theta_a}\right)_{ij}\phi_j$
  - $i, j = 1, \dots, M$ and $a = 1, \dots, (N^2 - 1)$
  - T_a – generators of the $SU(N)$ algebra ($M{\times}M$ matrices)

$$[T_a, T_b] = if_{abc}T_c$$

- non-Abelian symmetry transformation defined for each multiplet of fields separately
- $SU(N)$ *invariance of* $\mathcal{L}$: each term must consist of a product of fields such that the various representations are contracted into a singlet of the symmetry group

Non-Abelian symmetries

- If the symmetry group is not simple, we can consider an *independent rotation* within each simple subgroup
- Consider and $SU(3)\times SU(2)$ and a field ϕ that is a triplet under $SU(3)$ and doublet under $SU(2)$
  - $\phi_{\alpha i}$ with $\alpha = 1, 2, 3$ is the $SU(3)$-triplet index and $i = 1, 2$ is the $SU(2)$ doublet index
- Separate transformation law for $SU(3)$ and $SU(2)$:

$$\phi_{\alpha i} \to \left(e^{(i/2)\lambda_a \theta_a}\right)_{\alpha\beta} \phi_{\beta i}, \qquad \phi_{\alpha i} \to \left(e^{(i/2)\tau_b \theta_b}\right)_{ij} \phi_{\alpha j},$$

- λ_a – eight 3×3 Gell-Mann matrices such that $\lambda_a/2$ are the $SU(3)$ generators of the triplet representation
- τ_b – three 2×2 Pauli matrices such that $\tau_b/2$ are the $SU(2)$ generators of the doublet representation
- **Notation**: $SU(3)\times SU(2)\times U(1)$ symmetry (mixed Abelian and non-Abelian)
  - we denote a field ϕ is a $SU(3)$-triplet, $SU(2)$-doublet and carries a $U(1)$ charge of +1/6: $\phi(3, 2)_{+1/6}$

Non-Abelian global symmetries

- The product of the various representations must be contracted into a singlet of the imposed symmetry
- **Example:** Scalars and $SO(N)$
  - Global $SO(N)$ symmetry ($N \geq 2$)
  - Single scalar field in the fundamental representation, $\phi(N)$ (N scalar degrees of freedom)
  - There are no fermions
- The most general renormalizable $\mathcal{L}$ is

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi^\dagger \partial_\mu \phi - \frac{1}{2}m^2\phi^\dagger\phi - \frac{1}{4}\lambda\left(\phi^\dagger\phi\right)^2$$

- $\phi^\dagger\phi$ is the contraction of $(\bar{N})\times N$ into a singlet of $SO(N)$ – explicitly the contraction can be written as $\phi_i^*\phi_i$
- There is no internal symmetry which one could impose to set $m^2 = 0$ or $\lambda = 0$
- In the case of $N = 2$ the symmetry is Abelian and the model is the same as the one discussed last week

Non-Abelian global symmetries

- **Example:** Vectorial fermions and $U(N)$
  - Global $U(N) = SU(N)\times U(1)$ symmetry ($N \geq 2$)
  - Two fermion fields in the fundamental representation: $\psi_L(N)_{+1}, \psi_R(N)_{+1}$ ($4N$ degrees of freedom)
  - There are no scalars
- The most general renormalizable $\mathcal{L}$ is

$$\mathcal{L} = i\overline{\psi_L}\gamma^\mu\partial_\mu\psi_L + i\overline{\psi_R}\gamma^\mu\partial_\mu\psi_R - \left(m\overline{\psi_L}\psi_R + \text{h.c.}\right)$$

- We can't write Majorana terms since the fermions are charged under $U(1)$ (Dirac mass terms are allowed)
- Since the model is vectorial, we can combine them into a Dirac fermion ψ and rewrite $\mathcal{L}$ as

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

Non-Abelian global symmetries

- **Example:** Chiral fermions and $U(N)\times U(N)$
  - Global $U(N)_L\times U(N)_R = SU(N)_L\times SU(N)_R\times U(1)_L\times U(1)_R$ symmetry ($N \geq 2$)
  - Two fermion field: $\psi_L(N,1)_{1,0}, \psi_R(1,N)_{0,1}$
  - There are no scalars
- The most general renormalizable $\mathcal{L}$ is

$$\mathcal{L} = i\overline{\psi_L}\gamma^\mu\partial_\mu\psi_L + i\overline{\psi_R}\gamma^\mu\partial_\mu\psi_R$$

- Fermion mass terms can be forbidden by a symmetry (unlike scalar mass terms)
- The mass term vanishes because the fermion fields of the model are in a chiral representation of the symmetry

Non-Abelian local symmetries

- Non-Abelian local symmetries are commonly called "Yang-Mills theories"
- The following applies for both Abelian and non-Abelian local symmetries
  - terms that depend on scalar and/or fermion fields, but not on their derivatives and which are invariant under a global symmetry, are also invariant under the corresponding local symmetry
  - the kinetic terms are not invariant under the local symmetry
- To achieve invariance under a local non-Abelian symmetry (we will consider $SU(N)$) we need to add gauge fields to replace the derivative $\partial^\mu \phi$ with a covariant derivative $D^\mu \phi$ such that $D^\mu \phi$ and ϕ transform in the same way

$$\phi \to e^{iT_a\theta_a(x)}\phi, \qquad D^\mu \phi \to e^{iT_a\theta_a(x)} D^\mu \phi$$

- T_a's are the $N^2 - 1$ generators of the $SU(N)$ algebra and, for ϕ in an M −dimensional representation they are represented by $M \times M$ matrices
- The gauge fields that we introduce must restore the local symmetry for $N^2 - 1$ independent rotations

Non-Abelian local symmetries

$$\phi \to e^{iT_a\theta_a(x)}\phi, \qquad D^\mu\phi \to e^{iT_a\theta_a(x)}D^\mu\phi$$

- The covariant derivative is given by $D^\mu = \partial^\mu + igT_aG_a^\mu$ (g – dimensionless positive coupling constant)

- The transformation law for G_a^μ is given by (see question 4.6)

$$G_a^\mu \to G_a^\mu - f_{abc}\theta_b G_c^\mu - \frac{1}{g}\partial^\mu\theta_a$$

- Unlike the Abelian case the gauge boson is charged under the symmetry → self-interactions of the gauge fields

- We define the field strength $G_a^{\mu\nu}$ and introduce a kinetic term

$$[D^\mu, D^\nu] = igT_aG_a^{\mu\nu}, \qquad G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - gf_{abc}G_b^\mu G_c^\nu$$

$$\mathcal{L}_V = -\frac{1}{4}G_a^{\mu\nu}G_{a\mu\nu}$$

- Kinetic term is invariant under gauge transformations but a mass term $1/2m^2G_a^\mu G_{a\mu}$ is not

Summary: Abelian vs Non-Abelian symmetries

	Abelian	Non-Abelian
Field	Φ_q	$\Phi(n) \equiv \Phi_i, \quad i = 1, \dots, n$
Transformation	$\Phi \to e^{iq\theta}\Phi$	$\Phi_i \to \left(e^{iT_a\theta_a}\right)_{ij} \Phi_j$
	with q a real number	with T_a in the irrep of Φ
The gauge field	A_μ	G^a_μ
irrep of the gauge field	$q = 0$	Adjoint
Covariant derivative	$D^\mu = \partial^\mu + igqA^\mu$	$D^\mu = \partial^\mu + igT_a G^\mu_a$
Field strength tensor	$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$	$G^{\mu\nu}_a = \partial^\mu G^\nu_a - \partial^\nu G^\mu_a - g f_{abc} G^\mu_b G^\nu_c$
Gauge kinetic term	$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$	$-\frac{1}{4}G_{\mu\nu a}G^{\mu\nu}_a$
Gauge boson self-interactions	No	Yes

Spontaneous symmetry breaking

- Spontaneously broken symmetries play an important role in physics, and particle physics in particular
- Broken symmetry? Is there a difference between a broken symmetry and having no symmetry at all?
- The idea of a broken symmetry is meaningful in two scenarios:
  - Explicit breaking of a symmetry by a small parameter: $\mathcal{L}$ includes terms that break the symmetry, but these terms are characterized by a small parameter (dimensionless coupling or ratio between mass scales)
  - Spontaneously symmetry breaking (SSB): $\mathcal{L}$ is symmetric, but the vacuum state is not. Even though with SSB the symmetry is hidden, the number of parameters is the same as in the case of unbroken symmetry. The predictive power of a spontaneously broken symmetry is as strong as that of an unbroken symmetry

Global discrete symmetries: Z_2

- Model with an imposed Z_2 symmetry with a single scalar field ϕ, odd under the symmetry

$$\phi \to -\phi$$

- ϕ −parity symmetry of $\mathcal{L}$ (ϕ^3 term is forbidden)

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi)(\partial_\mu \phi) - \frac{\mu^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4$$

- $\mathcal{L}$ must be Hermitian: μ^2 and λ must be real and we must have $\lambda > 0$

- μ^2 can be both positive and negative but we will consider here the case $\mu^2 < 0$

$$\frac{\partial V}{\partial \phi} = \phi(\mu^2 + \lambda\phi^2) = 0$$

- Two possible minima of the potential:

$$\phi_\pm = \pm\sqrt{-\frac{\mu^2}{\lambda}} \equiv \pm v$$

Global discrete symmetries: $\boldsymbol{Z_2}$

- The classical solution would be either ϕ_+ or ϕ_-
- ϕ acquires a vacuum expectation value (VEV):

$$\langle\phi\rangle \equiv \langle 0|\phi|0\rangle \neq 0$$

- Perturbative calculations should involve expansion around the classical minimum
- The two solutions are physically equivalent: physics cannot depend on our choice
- Let's expand around ϕ_+ and define ϕ'

$$\phi' = \phi - v \qquad \Longrightarrow \qquad \mathcal{L} = \frac{1}{2}(\partial^\mu\phi')(\partial_\mu\phi') - \frac{1}{2}(2\lambda v^2)\phi'^2 - \lambda v\phi'^3 - \frac{\lambda}{4}\phi'^4$$

- We used $\mu^2 = -\lambda v^2$
- Compare with the most general Lagrangian

$$\mathcal{L}_S = \frac{1}{2}(\partial^\mu\phi')(\partial_\mu\phi') - \frac{m^2}{2}\phi'^2 - \frac{\eta}{2\sqrt{2}}\phi'^3 - \frac{\lambda}{4}\phi'^4$$

Global discrete symmetries: $\boldsymbol{Z_2}$

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi')(\partial_\mu \phi') - \frac{1}{2}(2\lambda v^2)\phi'^2 - \lambda v \phi'^3 - \frac{\lambda}{4}\phi'^4$$

<u>Important points:</u>

- $\mathcal{L}$ includes all possible terms for a real scalar field and has no ϕ' parity symmetry. The symmetry is hidden and spontaneously broken by our choice of ground state $\langle\phi\rangle = +v$
- Yet $\mathcal{L}$ is not the most general renormalizable one for a scalar field. **Why?**

Global discrete symmetries: $\boldsymbol{Z_2}$

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi')(\partial_\mu \phi') - \frac{1}{2}(2\lambda v^2)\phi'^2 - \lambda v \phi'^3 - \frac{\lambda}{4}\phi'^4$$

Important points:

- $\mathcal{L}$ includes all possible terms for a real scalar field and has no ϕ' parity symmetry. The symmetry is hidden and spontaneously broken by our choice of ground state $\langle \phi \rangle = +v$
- Yet $\mathcal{L}$ is not the most general renormalizable one for a scalar field: depends only on two parameter and not three
- $\eta^2 = 4\lambda m^2$ – clue that the symmetry is spontaneously rather than explicitly broken!
- We can choose any two parameters to describe the system: SSB does not introduce new parameters!
- SSB changes dimensionful parameters but not dimensionless ones
- The model describes a particle with a scalar particle of mass $2\lambda v^2 = -2\mu^2$ (excitation of the ϕ' field)

Global Abelian continuous symmetries: $\boldsymbol{U(1)}$

- Model with an imposed $U(1)$ symmetry and a single scalar field ϕ with $q = +1$

$$\phi \to e^{i\theta}\phi \qquad\qquad \mathcal{L} = (\partial^\mu \phi^\dagger)(\partial_\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$

- Equivalently we can impose an $SO(2)$ symmtery

$$\begin{pmatrix} \phi_R \\ \phi_I \end{pmatrix} \to O \begin{pmatrix} \phi_R \\ \phi_I \end{pmatrix}, \qquad O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_R)(\partial^\mu \phi_R) + \frac{1}{2}(\partial_\mu \phi_I)(\partial^\mu \phi_I) - \frac{\mu^2}{2}(\phi_R^2 + \phi_I^2) - \frac{\lambda}{4}(\phi_R^2 + \phi_I^2)^2$$

- μ^2 and λ must be real and we must have $\lambda > 0$ and. We consider $\mu^2 < 0$ and define $v^2 = -\mu^2/\lambda$

$$V = \lambda\left(\phi^\dagger \phi - \frac{v^2}{2}\right)^2 \qquad \Longrightarrow \qquad 2\langle \phi^\dagger \phi \rangle = \langle \phi_R^2 + \phi_I^2 \rangle = v^2 = -\frac{\mu^2}{\lambda}$$

- Circle of radius v in the (ϕ_R, ϕ_I) that corresponds to the minimum of the potential

Global Abelian continuous symmetries: $\boldsymbol{U(1)}$

- We have to choose a specific vacuum to expand around, and we choose only the real component of ϕ to carry VEV

$$\langle \phi_R \rangle = v, \qquad \langle \phi_I \rangle = 0$$

- We define the real scalar fields with vanishing VEVs

$$h = \phi_R - v, \qquad \xi = \phi_I, \qquad \langle h \rangle = \langle \xi \rangle = 0$$

$$\Downarrow$$

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu h\right)(\partial^\mu h) + \frac{1}{2}\left(\partial_\mu \xi\right)(\partial^\mu \xi) - \lambda v^2 h^2 - \lambda v h(h^2 + \xi^2) - \frac{\lambda}{4}(h^2 + \xi^2)^2$$

Global Abelian continuous symmetries: $\boldsymbol{U(1)}$

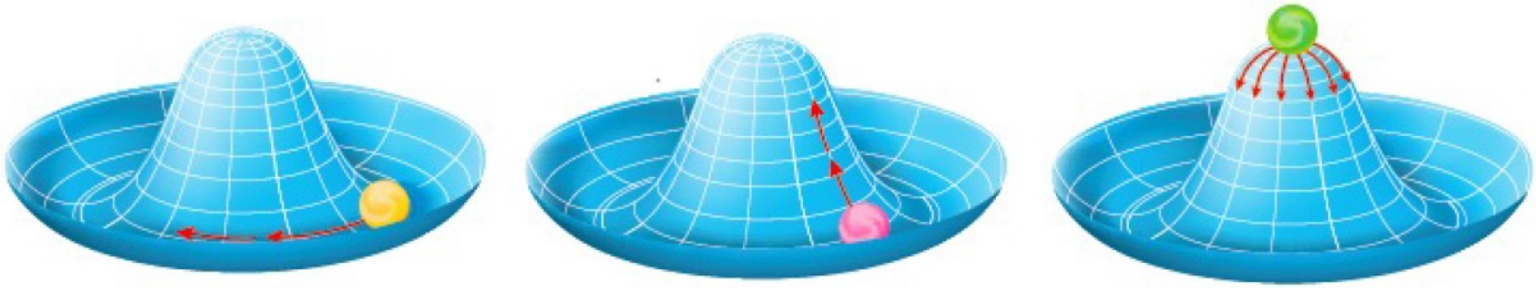
$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) - \lambda v^2 h^2 - \lambda v h(h^2 + \xi^2) - \frac{\lambda}{4}(h^2 + \xi^2)^2$$

Important points:

- Spontaneously broken $SO(2)$ symmetry: presence of the $h(h^2 + \xi^2)$ term
- $\mathcal{L}$ describes one massive scalar, h, with $m^2 = 2\lambda v^2$ and one massless boson ξ
- If the symmetry was not broken, we wouldn't be able to distinguish the two components of the complex scalar field, which after SSB have different masses
- Only two independent parameters as for a $\mathcal{L}$ with an unbroken $SO(2)$
- Quartic terms, with dimensionless couplings, are the same as before SSB (only dimensionful couplings are modified)
- Arbitrary choice to assign the VEV to the real component of ϕ (physics doesn't depend on this choice)
- We write VEV as $\langle\phi_R\rangle = v$ or equivalently $\langle\phi\rangle = v/\sqrt{2}$ (factor $\sqrt{2}$ when we move between real and complex fields)

Global Abelian continuous symmetries: $\boldsymbol{U(1)}$

$$V = \lambda\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2$$



$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) - \lambda v^2 h^2 - \lambda v h(h^2 + \xi^2) - \frac{\lambda}{4}(h^2 + \xi^2)^2$$

Interesting features of our model:

- Existence of a massless scalar field ξ
- Not specific to our model, but rather the result of a general theorem called Goldstone's theorem:
  - SSB of a global continuous symmetry is accompanied by massless scalars
  - The number of the massless scalars and their quantum numbers equal those of the broken generators
  - The massless scalars are called Nambu-Goldstone Bosons
- SSB is possible only if the vacuum is degenerate (for continuous symmetry it is also continuous)
- In one direction the potential is flat corresponding to a massless DoF

Fermion masses

- SSB can give masses to chiral fermions
- Let's consider $U(1)$ symmetry with a left-handed fermion ψ_L, a right-handed fermion ψ_R, and a complex scalar ϕ

$$q(\psi_L) = +1, \qquad q(\psi_R) = +2, \qquad q(\phi) = +1$$

$$\mathcal{L} = \mathcal{L}_{\text{kin}} - \mu^2 \phi^\dagger \phi - \lambda\left(\phi^\dagger \phi\right)^2 - (Y\phi\overline{\psi_R}\psi_L + \text{h.c.})$$

- We take $\mu^2 < 0$ so we get the potential from slide 18, leading to a VEV for ϕ: $|\langle\phi\rangle| = v/\sqrt{2} \neq 0$ and we choose

$$\langle\phi_R\rangle = v, \qquad \langle\phi_I\rangle = 0$$

- We define the real fields h and ξ such that they have vanishing VEV

$$\phi = \frac{h + v + i\xi}{\sqrt{2}}$$

Fermion masses

- We define the real fields h and ξ such that they have vanishing VEV

$$\phi = \frac{h + v + i\xi}{\sqrt{2}}$$

- Expanding around the chosen vacuum we find

$$\mathcal{L} = \mathcal{L}_{\text{kin}} - V(h,\xi) - \left(\frac{Yv}{\sqrt{2}}\overline{\psi_R}\psi_L + \frac{Y}{\sqrt{2}}(h + i\xi)\overline{\psi_R}\psi_L + \text{h.c.}\right)$$

- ψ_L and ψ_R combine to form a Dirac fermion with mass

$$m_\psi = \frac{Yv}{\sqrt{2}}$$

- Possible because the symmetry under which the fermion is chiral is broken!

- In the more general case, the symmetry might only be partially broken (only a subgroup of the original group)

- In this case the conditions for generating fermion masses are

  - *Dirac mass:* fermion representation is vector-like under the unbroken subgroup

  - *Majorana mass:* fermion is neutral under unbroken $U(1)$ groups + in real representation of unbroken non-Abelian subgroups

Local symmetries: the Higgs mechanism

- Let's consider SSB of a local $U(1)$ symmetry and a single complex scalar field

$$\phi \to e^{i\theta(x)}\phi \qquad \mathcal{L} = (D^\mu\phi)^\dagger(D_\mu\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2$$

- Covariant derivative is defined by

$$D^\mu\phi = (\partial^\mu + igA^\mu)\phi$$

- We consider the case of $\mu^2 < 0$, leading to SSB via a VEV of ϕ

$$\langle\phi\rangle = \frac{v}{\sqrt{2}}, \qquad v^2 = -\frac{\mu^2}{\lambda}$$

- We again choose the real component of ϕ to carry the VEV
- We write the complex scalar in terms of the two real scalar fields with vanishing VEVs, $\langle h\rangle = \langle\xi\rangle = 0$

$$\phi(x) = e^{i\xi(x)/v}\,\frac{v + h(x)}{\sqrt{2}}$$

Local symmetries: the Higgs mechanism

- The symmetry is spontaneously broken and we write $\mathcal{L}$ in terms of the VEV-less fields h and ξ
- $\mathcal{L}$ no longer invariant under the broken symmetry transformation. The transformation constitutes a change of basis
- We can choose a basis by choosing a specific gauge: $\theta(x) = -\xi(x)/v$ (unitary gauge)

$$\phi \rightarrow \phi' = \frac{h+v}{\sqrt{2}}, \qquad A_\mu \rightarrow V_\mu = A_\mu + \frac{1}{gv}\partial_\mu \xi$$

- ϕ' has one DoF and V_μ has three

$$\mathcal{L} = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}\left(\partial_\mu h\right)(\partial^\mu h) + \frac{g^2v^2}{2}V_\mu V^\mu - \frac{2\lambda v^2}{2}h^2 + \frac{g^2}{2}V_\mu V^\mu h(2v+h) - \lambda v h^3 - \frac{\lambda}{4}h^4$$

- The kinetic term of the gauge boson is independent of the gauge fixing

$$\partial_\mu V_\nu - \partial_\nu V_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Local symmetries: the Higgs mechanism

$$\mathcal{L} = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}\left(\partial_\mu h\right)(\partial^\mu h) + \frac{g^2v^2}{2}V_\mu V^\mu - \frac{2\lambda v^2}{2}h^2 + \frac{g^2}{2}V_\mu V^\mu h(2v+h) - \lambda v h^3 - \frac{\lambda}{4}h^4$$

Important points:

- the model consists of a massive vector boson of mass $m_V^2 = (gv)^2$ and a massive scalar of mass-square $m_h^2 = 2\lambda v^2$
- the sign of the mass-squared term is opposite for a vector boson and a scalar
- h scalar is called "a Higgs boson" and the related field, which acquires a VEV (ϕ) is called the Higgs field
- the source of the mass-squared term for the vector bosons is the kinetic term of the Higgs field
- the propagator of a massive gauge boson depends on the gauge choice
- the ξ field is "eaten" to give mass to the gauge boson: convenient choice to make the phase to be the "eaten" DoF
- in the limit $g \to 0$ we have $m_V \to 0$: massless gauge boson and a massless scalar

Local symmetries: the Higgs mechanism

$$\mathcal{L} = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}\left(\partial_\mu h\right)(\partial^\mu h) + \frac{g^2v^2}{2}V_\mu V^\mu - \frac{2\lambda v^2}{2}h^2 + \frac{g^2}{2}V_\mu V^\mu h(2v + h) - \lambda v h^3 - \frac{\lambda}{4}h^4$$

<u>**Interactions:**</u>

- hVV coupling is proportional to the mass-squared of the vector boson
- the dimensionless $VVhh$ and $hhhh$ couplings are unchanged from the symmetric Lagrangian
- the unbroken $\mathcal{L}$ (slide 24) depends on three parameters, which can be taken to be g, v, and λ
- $\mathcal{L}$ after SSB has two mass terms and four interaction terms which depend on the same three parameters

The Higgs mechanism summary

Type	Consequences
Spacetime	Conservation of energy, momentum, angular momentum
Discrete	Selection rules
Global (exact)	Conserved charges
Global (spon. broken)	Massless scalars
Local (exact)	Interactions, massless spin-1 mediators
Local (spon. broken)	Interactions, massive spin-1 mediators

- SSB gives masses to the gauge bosons related to the broken generators
- Gauge bosons of an unbroken symmetry remain massless: their masslessness is protected by a symmetry
- The field that acquires a VEV must be a scalar field (otherwise its VEV would break Lorentz invariance)
- SSB of a symmetry can give masses also to fermions via Yukawa interactions
- States with different QNs under the broken symmetry but the same QNs under the unbroken subgroup can mix!

Summary of Lecture 3

Main learning outcomes

- Examples and characteristics of non-Abelian symmetries (global and local) and how they give rise to vector boson self-interactions
- Mechanism of spontaneous symmetry breaking, generating masses of scalar, vector, and fermions fields